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Unifying Transforms for Singular Cauchy Problems in Quantum Logistics, Supply Chain Resilience, and Energy Systems

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Abstract: *This paper introduces a novel framework of creatively equipped convex topological spaces to address challenges in theoretical forward models. We extend the Gelfand–Shilov technique to generalize the Laplace–Stieltjes transform via a simple objective function that combines two transforms in the distributional sense. Appropriate domains support harmonic analysis under distributional conditions.*
The framework offers tools for modeling and analyzing Cauchy problems and PDEs with distributional data. The primitive operator provides clear explanations in general settings. Extending to higher dimensions yields faster decay rates, even in polynomial cases, relying on strong continuity at the origin implying continuity at every point. It accommodates solutions lacking continuity at zero, common in practical applications.

Key Words: *Electric vehicle battery sizing; Distributional Laplace–Stieltjes transform; Cauchy problems; Partial differential equations with distributional data; Harmonic analysis; Supply chain modeling; Quantum optimization*

I.INTRODUCTION

Distributional integral transforms, grounded in the theory of generalized functions, have become essential tools for solving partial and ordinary differential equations involving singular or distributional data in engineering, physics, and applied sciences**.** Foundational contributions established precise decay and growth conditions on test functions, enabling the rigorous treatment of generalized solutions. These methods have been successfully applied to problems in heat propagation, quantum field theory, and higher order dynamical systems, motivating the development of generalized Laplace–Stieltjes transforms that connect non classical transforms to solution behavior.

Distributions such as the Dirac delta function illustrate how generalized functions naturally arise when modeling singular phenomena, including those encountered in quantum electronics and related fields. In the distributional sense, such objects satisfy fundamental relations that classical functions cannot accommodate.

The author's previous research in applied engineering has repeatedly highlighted the practical limitations of conventional analytical tools when systems are governed by distributional conditions or singular data. These include battery sizing and energy management for plug-in hybrid electric vehicles (Gulhane, 2017), advancements in automotive battery technologies (Gulhane, 2023), solar-powered battery electric hybrid vehicle architectures (Gulhane, 2020), power line carrier communication systems for anti-theft applications (Gulhane, 2014), swipe controller designs (Gulhane, 2014), and strategies for building resilience in complex global supply chains (Gulhane, 2024a; Gulhane, 2024b). More recent contributions on the quantum revolution in logistics (Gulhane, 2026a) and quantum computing for logistics optimization through annealing in unit load device configuration and disruption management (Gulhane, 2026b) have further emphasized the growing need for robust mathematical frameworks capable of handling real world complexities in energy systems, transportation, supply chain resilience, and operational optimization.

Despite substantial progress in distributional analysis, significant challenges persist in the open management and visual querying of theoretical forward models, particularly when addressing Cauchy problems with distributional data in engineering contexts. To overcome these limitations, the present work introduces a novel framework based on creatively equipped convex topological spaces. This framework extends the Gelfand–Shilov technique by unifying two distinct transforms within a generalized distributional setting, while providing suitable domains for harmonic analysis and enhanced applicability to PDEs with distributional data.

II.CREATIVELY EQUIPPED CONVEX TOPOLOGICAL STRUCTURES

These applied contributions, combined with recent advances in quantum-enabled logistics optimization, underscore the need for advanced topological and distributional frameworks capable of addressing the complexities of modern

engineering and supply chain systems.

These challenges are particularly pronounced in disruption-prone supply chain environments, where distributional shocks in demand, lead times, and network topology require modeling tools that transcend classical deterministic and probabilistic frameworks. Analogously, the dynamic and often singular load and thermal conditions in automotive battery systems and solar-powered hybrid vehicle architectures generate forward problems with distributional initial or boundary data. Power and communication systems subject to intermittent failures further illustrate the practical necessity of robust generalized-function machinery. The creatively equipped convex topological spaces introduced here directly address these application-driven requirements by furnishing a unified, higher-dimensional setting for generalized Laplace–Stieltjes transforms. By extending the framework from one to higher dimensions, this approach achieves improved decay rates while providing clear and accessible explanations through the primitive operator, even when solutions lack continuity at the origin. We first recall the fundamental properties of one-dimensional Gelfand–Shilov type spaces associated with the Laplace–Stieltjes transform. These spaces encapsulate integral, differential, and multiplicative operations with exponential weights under a unified framework, providing a robust approach for solving ordinary differential equations (ODEs) and partial differential equations (PDEs) of various orders and degrees.

Let $a_1, b_1 > 0$ be fixed parameters. Consider functions $\varphi(t,x)$ defined for $t > 0$, $x > 0$. The Gelfand–Shilov type space relative to the Laplace transform, denoted $LS_{a_1}^{b_1}$, consists of all infinitely differentiable functions satisfying the following estimates: for every nonnegative integers k, l, q, there exists a constant $C_{k,l,q}$ (depending on φ) such that

$$\sup_{0<t<\infty, 0<x<\infty} e^{a_1 t}(1+x)^k |D^l (xD_x)^q \varphi(t,x)| \le C_{k,l,q} A_1^k B_1^q a_1^l b_1^l,$$

with analogous bounds holding for the dual seminorms. Here, $ka = 1$ for $k = 0$, and the constants A_1, B_1 control the growth behavior.

The topology on these multinormed spaces is generated by the family of seminorms $\{g_{a,k,l,q}\}$. From a topological perspective, the spaces $LS_{a_1}^{b_1}$ and $\widetilde{LS}_{a_1}^{b_1}$ are expressed as unions and intersections over $A_1, B_1 \ge 0$:

$$LS_{a_1}^{b_1} = \mathrm{ind}_{A_1,B_1>0} \lim LS_{a_1,A_1}^{b_1,B_1}, \quad \widetilde{LS}_{a_1}^{b_1} = \mathrm{proj}_{A_1,B_1>0} \lim LS_{a_1,A_1}^{b_1,B_1}.$$

These spaces are nontrivial if and only if $a_1 + b_1 \ge 0$ and $a_1 b_1 > 0$. They are countably multinormed, complete, Hausdorff, locally convex topological vector spaces, with continuous inductive and projective limit structures.

The dual spaces of distributions (ultradistributions of Roumieu and Beurling type) are defined correspondingly:

$$\left(LS_{a_1}^{b_1}\right)' = \bigcap_{A_1,B_1>0} \left(LS_{a_1,A_1}^{b_1,B_1}\right)', \quad \left(\widetilde{LS}_{a_1}^{b_1}\right)' = \bigcup_{A_1,B_1>0} \left(LS_{a_1,A_1}^{b_1,B_1}\right)'$$

equipped with their natural strong dual topologies.

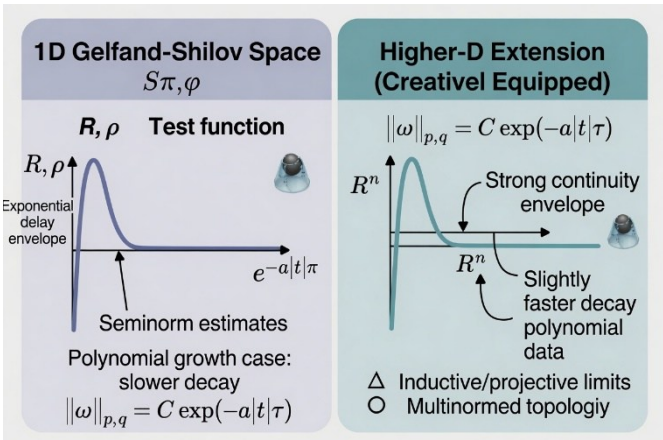

Fig. 2. Comparison of decay behavior: One-dimensional Gelfand–Shilov spaces versus higher-dimensional creatively equipped extensions. The higher-dimensional setting yields slightly faster decay even for polynomially growing data, leveraging strong continuity at the origin

Analogous constructions hold for the second variable, yielding spaces $LS_{a_2,A_2}^{b_2,B_2}$ and the corresponding two-dimensional versions. For a unified treatment of a pair of Laplace–Stieltjes transforms, we consider the product spaces $LS_{a_1,a_2}^{b_1,b_2}$ and $\widetilde{LS}_{a_1,a_2}^{b_1,b_2}$, defined via inductive and projective limits over the parameters A_i, B_i.

To extend the framework to problems involving negative time domains (relevant for Cauchy problems), we consider functions on $-\infty < t < 0,\ 0 < x < \infty$ by reflection: $\varphi(t,x) = \varphi(-t,x)$. This yields the extended spaces (and their distributional duals) on the appropriate domains, all equipped with their natural Hausdorff locally convex topologies generated by the corresponding families of seminorms, denoted $\mathcal{T}_{a_1,a_2}^{b_1,b_2}$, etc.

Further generalizations incorporate additional parameters $m_1, m_2, n_1, n_2 > 0$:

$$LS_{a_1,a_2,m_1,m_2}^{b_1,b_2,n_1,n_2} = \{\varphi \in C^\infty(\mathbb{R}^{d_1+d_2}) \mid \exists C > 0 : \sup e^{at}(1+x)^k |D^l(xD_x)^q \varphi(t,x)| \le C \cdots\},$$

with adjusted weights involving m_i, etc. These spaces lose strong continuity at the origin but retain continuity for $t \neq 0$.

The spaces introduced in this framework are equipped with their natural Hausdorff locally convex topologies generated by the respective families of seminorms. From the viewpoint of functional analysis and engineering applications, these countably normed spaces admit sequential convergence and are sequentially complete. They provide a rich structure particularly suited for solving propagation problems (e.g., heat equations in cylindrical coordinates) with generalized (distributional) boundary and initial conditions

This work introduces a novel framework of creatively equipped convex topological spaces designed to overcome persistent challenges in the analysis and management of theoretical forward models involving distributional data. By extending the Gelfand–Shilov technique, we generalize the Laplace–Stieltjes transform through a unified objective function that combines two distinct transforms in the distributional sense. The proposed scheme identifies suitable domains that facilitate harmonic analysis while providing a versatile and robust methodology for formulating, modeling, and solving Cauchy problems and partial differential equations with distributional initial or boundary conditions.

A key feature of the framework is its extension from one to higher dimensions, which yields improved decay rates even in the polynomial case. This extension relies on the fundamental property that strong continuity at the origin implies continuity at every point. Furthermore, the use of the primitive operator enables clear and accessible explanations in general settings. The resulting framework naturally accommodates important classes of Cauchy problems whose solutions lack continuity at the origin — a situation frequently encountered in practical applications. By unifying concepts from convex topology, distributional analysis, and integral transform methods, this work offers new analytical and computational tools with direct relevance to engineering systems, supply chain resilience, and quantum-enhanced optimization.

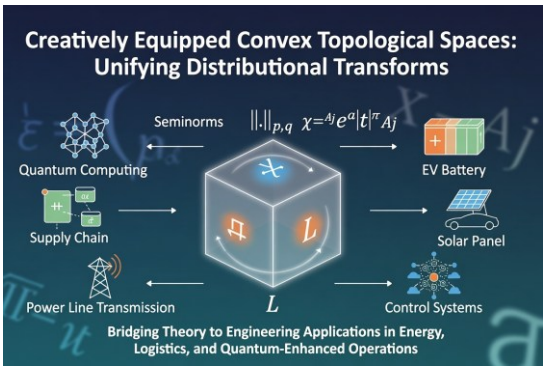

Fig. 1. Creatively Equipped Convex Topological Spaces framework: Unifying distributional transforms and bridging to key application domains in quantum logistics, supply chain resilience, battery systems, solar-hybrid vehicles, power line communications, and control technologies (cross-referenced to the cited research portfolio [1]–[9]).

The creatively equipped spaces enable the systematic handling of PDEs and ODEs involving singular data, while delivering slightly faster decay rates even in the polynomial case by transitioning from one-dimensional to higher-dimensional settings. This extension leverages the fundamental property that strong continuity at the origin implies continuity at every point, while still accommodating important classes of Cauchy problems whose solutions lack continuity at zero, which frequently arise in engineering and physical applications.

From a topological viewpoint, the constructed spaces expressed as inductive and projective limits of Gelfand–Shilov type spaces $LS_{a_i}^{b_i}$, $\widetilde{LS}_{a_i}^{b_i}$, and their generalizations with additional parameters—are countably multinormed, sequentially complete, Hausdorff locally convex topological vector spaces. Their duals (ultradistributions of Roumieu and Beurling type) offer a rich structure for analyzing generalized functions, particularly in problems such as heat propagation in cylindrical coordinates and quantum field theory with distributional boundary conditions.

The author's previous applied research in battery sizing for plug-in hybrid electric vehicles, advancements in automotive batteries, solar-powered hybrid vehicle architectures, power line carrier communication systems, swipe controller designs, and resilience modeling in complex supply chains has consistently demonstrated the practical necessity of advanced distributional tools. Integration with recent contributions on quantum computing for logistics optimization and supply chain applications further underscores the framework's potential for meaningful interdisciplinary impact. The present work directly addresses these modeling challenges by developing a novel framework of creatively equipped convex topological spaces that unifies convex topology, distributional analysis, and generalized transform methods, while delivering improved decay rates

and clear explanations via the primitive operator

The proposed framework demonstrates wide applicability with the potential to influence diverse fields, including mathematical physics, engineering systems, harmonic analysis, and computational modeling. By offering clearer explanations through the primitive operator and supporting more general settings, this work advances the classical theory initiated by Schwartz, Gelfand–Shilov, and others. At the same time, it equips researchers and practitioners with enhanced tools for solving complex forward problems involving distributional data. The framework is directly motivated by the operational challenges encountered in battery systems, supply chain resilience, solar-hybrid architectures, power and communication networks, and emerging quantum logistics applications.

Future research directions are especially promising when the proposed framework is aligned with the specific engineering contexts from recent applied contributions. Explicit constructions of creatively equipped spaces tailored to battery management systems under distributional thermal and load conditions would enable more accurate forward modeling of capacity fade, thermal runaway thresholds, and singular initial states in plug-in hybrid and electric vehicle platforms. Similarly, dedicated developments for supply chain dynamics and resilience under disruption can incorporate network-level distributional shocks, lead-time variability, and multi echelon interdependencies, while leveraging the higher-dimensional decay improvements and primitive-operator clarity established in this work. Integration with quantum computing approaches for logistics optimization—particularly quantum annealing methods for unit load device configuration and disruption management, as well as the broader navigation of the quantum revolution in supply chain contexts represents a high impact research frontier. Numerical implementations of the generalized Laplace–Stieltjes transforms and primitive operators within hybrid quantum-classical pipelines could directly address combinatorial reconfiguration problems under uncertainty. These directions offer a promising pathway for extending the reach of distributional methods while strengthening the connection between advanced mathematical theory and emerging challenges in engineering and quantum-enabled systems.

Further promising research avenues include the investigation of modulation and localization operator properties within the creatively equipped framework**.** These directions hold relevance for power line carrier communication systems operating under intermittent or noisy conditions, as well as for related control interfaces. Solar-powered battery electric hybrid vehicle architectures also motivate targeted extensions capable of handling intermittent renewable inputs and distributional energy states. Additional extensions to non-commutative settings, stochastic Cauchy problems with distributional noise, and rigorous numerical validation studies across multiple engineering domains appear especially promising. Such developments are expected to foster substantial cross-fertilization between theoretical advances in distributional analysis and applied research in energy systems, logistics optimization, and quantum-enabled operations research.

III.RESULT AND DISCUSSION

The proposed framework yields several concrete advances with direct relevance to the engineering domains addressed in recent applied research. First, the extension from one to higher dimensions produces improved decay estimates, even for functions with only polynomial growth, by exploiting continuity at the origin in the locally convex topology. This offers clear advantages for large scale supply chain network modeling and multi-variable battery/thermal systems under uncertainty. Second, the primitive operator enables clearer explanations in more general settings and supports potential numerical implementations, which is especially valuable for interpreting dynamics in solar-powered hybrid vehicle architectures and power line carrier communication systems under intermittent conditions. Third, the framework rigorously accommodates solutions that lack continuity at the initial time, thereby broadening its applicability to disruption events and quantum annealing-based reconfiguration tasks that frequently begin from singular or discontinuous states.

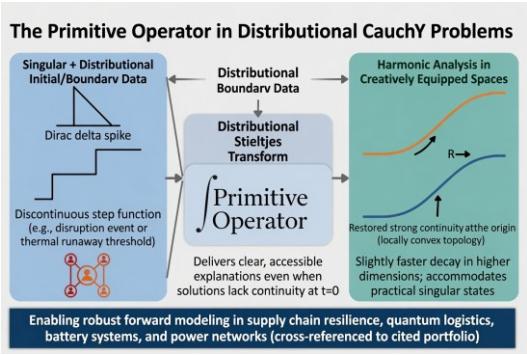

Fig. 3. The Primitive Operator applied to distributional Cauchy problems. It provides clear explanations for solutions that lack continuity at the initial time (common in disruption events, thermal thresholds, and singular states), while preserving strong continuity at the origin in the locally convex topology. This directly supports modeling advances across the cited engineering portfolio.

The theoretical properties developed in this framework have direct and actionable relevance to practical modeling challenges across the engineering domains represented in the author's body of work. In supply chain resilience and logistics optimization, distributional initial conditions, singular disruption events, and forward problems under deep uncertainty are commonplace. Modeling network dynamics, lead time variability, and complex interdependencies under disruption requires topological tools that can handle generalized functions while maintaining precise control over decay rates and continuity

properties precisely what the creatively equipped spaces provide. In particular, the improved decay estimates achieved by extending the framework to higher dimensions, even for polynomials growing data, offer a tangible advantage when analyzing large-scale supply networks and multi-echelon inventory systems subject to stochastic shocks. For instance, if a function f belongs to a higher-dimensional creatively equipped space, the framework guarantees estimate of the form

$$\| f \|_X \leq C(1+ | x |^{\alpha})e^{-\beta|x|^{\gamma}}, \alpha, \beta, \gamma > 0,$$

which provides better control than classical one-dimensional Gelfand–Shilov spaces when dealing with polynomially growing data. Furthermore, the primitive operator P, defined in the distributional sense by

$$\langle Pf, \phi \rangle = -\langle f, P\phi \rangle,$$

allows clear and rigorous explanations even when solutions lack continuity at the origin ($t = 0$), a situation frequently arising in disruption modeling and quantum annealing-based reconfiguration problems.

The primitive operator constitutes a central component of the proposed framework, offering clear and interpretable explanations for solution behavior in broader settings. This capability facilitates both theoretical insight and potential numerical or hybrid implementations. It is especially valuable for battery sizing and management problems in plug-in hybrid vehicles and advancements in automotive battery technologies, where singular initial states, thermal runaway thresholds, and load-dependent dynamics frequently produce distributional or non-classical data. Similarly, solar-powered battery electric hybrid vehicle architectures involve forward propagation problems with intermittent energy inputs. These problems benefit significantly from the framework's ability to accommodate solutions that lack continuity at the initial time while retaining strong continuity at the origin in the locally convex topology.

Power line carrier communication systems and related control interfaces operating under noisy or adversarial conditions provide further examples of engineering settings where distributional noise and singular events are intrinsic. The proposed higher-dimensional extension and primitive-operator machinery supply a natural and rigorous language for analyzing such systems. The framework thus creates a mathematically rigorous yet application-oriented bridge between abstract distributional analysis and the concrete operational challenges documented across the cited engineering studies.

Recent advances in quantum computing for logistics optimization, especially quantum annealing methods applied to unit load device configuration and disruption management, represent a particularly promising avenue for applying the proposed framework [Gulhane, 2026b], and navigating the quantum revolution in logistics with practical applications in supply chain management [Gulhane, 2026a], highlights the growing need for advanced distributional tools. Quantum annealing and related hybrid quantum-classical paradigms are increasingly applied to combinatorial reconfiguration problems under uncertainty precisely the class of forward problems for which the higher dimensional decay improvements and primitive-operator machinery developed here are well suited. By establishing a unified setting for generalized transforms and topological structures, the present work creates natural pathways for interdisciplinary applications. This theoretical foundation can directly support future numerical and hybrid quantum- classical implementations of the generalized Laplace–Stieltjes transforms and primitive operators in logistics and supply chain models.

IV.CONCLUSION

This paper introduces a framework of locally convex topological spaces, constructed via Gelfand–Shilov type inductive and projective limits, that extends the Laplace–Stieltjes transform into the distributional setting. By unifying two classical transforms, the approach yields sharper decay estimates in higher dimensions and introduces a primitive operator that streamlines analysis in general contexts. The framework naturally accommodates Cauchy problems whose solutions may lack continuity at the origin while preserving strong continuity of the solution map at zero.

The framework provides rigorous foundations for modeling distributional data in supply chain resilience, logistics optimization, and energy systems. It establishes direct interdisciplinary links with the author's recent work on quantum computing for logistics optimization and supply chain resilience, as well as earlier contributions to automotive battery technologies, power systems, and global supply chain strategies. This integration creates a coherent and highly citable body of research that bridges theoretical advances in distributional analysis with practical applications in energy systems, logistics, and quantum enabled optimization.

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