



# Contingency Analysis of Power System

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**Abstract:** Maintaining power system security is one of the challenging tasks for the power system engineers. Contingency analysis is one of the important aspects of power system security assessment. Various probable outages are analyzed in CA. It is being widely used to predict the effect of outages like failures of equipment, transmission line etc. and to take necessary actions to keep the power system secure and reliable. With the help of Fast Decoupled Load Flow (FDLF), the PIP and PIV have been calculated in MI-POWER environment and contingency ranking is made. Further the contingency selection has been done by using Radial Basis Function (RBF) Neural Network in MATLAB environment. The effectiveness of the method has been tested on IEEE-14 Bus test systems.

## I. INTRODUCTION

There Electricity is the indispensable form of energy in the modern societies; its demand has been increasing year by year. The system with generation of power, transmission of power and distribution of power with the control strategies is called electrical power system. Power system consists of various components namely generator, transformer, circuit breaker, conductor and load [1]. The failure of any one of the component during its normal operating condition leads to the violation of existing state of the system. The process of investigating whether the system is secure or not in a set of proposed contingencies is called contingency analysis or the effect of the line outage when the rest of the system is stable is called contingency study. The power system security plays an important role for the proper operation of the system. In the field of power system planning and co-ordination, the main task is to make the system to be secured and reliable. The power system network is monitored and controlled by the SCADA system. It helps the system operator to take the remedial actions before the network comes to unsecured state. When the specified operating limits are violated, the system comes to emergency state. This type of violation (Bus voltages and active power flow) leads to the contingencies. The role of the system operator is to resist the outcome of contingencies.

The contingency ranking is sorted according to the severity index or performance index [2]. Power system security mainly performs three functions. These functions are carried out in SCADA unit.

- System Monitoring
- Contingency Analysis

## II. METHODS OF CONTINGENCY ANALYSIS

There are various methods used for contingency analysis purpose. Methods based on AC power flow calculations are considered to be deterministic methods which are accurate compared to DC power flow methods. In deterministic methods line outages are simulated by actual removal of lines instead of modeling. AC power flow methods are accurate but they are computationally expensive and excessively demanding of computational time. Because contingency analysis is the only tool for detecting possible overloading conditions requiring the study by the power system planner computational speed and ease of detection are paramount considerations.

### 2.1 Contingency Analysis Using DC Power Flow

This method is based on DC power flow equation to simulate single or multiple contingencies. These equations are  $N-1$  in number, where  $N$  is the number of buses. In this method the line resistances are neglected, only real power flows are modeled ignoring the reactive power flows. This results in a linear model of the network to facilitate performing multiple contingency outages using the principle of super position.

Each transmission line is represented by its susceptance  $B_{ij}$ .

Impedance  $Z = r + jx$  ----- (2.1)

Inverse of impedance  $Y = G + jB$  -- (2.2)

$G = \frac{r}{r^2 + x^2} \approx 0$  ----- (2.3)

$$B = -x r 2 + x 2 \approx -1 x \text{ -----(2.4)}$$

In this method only the real part of the power flow equations are considered that is the effect of reactive power Q is neglected and all the bus voltages are assumed to be 1 pu. the matrix B' is computed on the basis that all the resistances are zero from equation 2.5

$$B'_{ik} = -B_{ij} = 1/x_{ij} \text{ -----(2.5)}$$

Where  $x_{ij}$  is the reactance of the line connecting buses i and j. The angles and real powers are solved by iterating Equation 2.6.

$$\delta = [B']^{-1} \Delta P \text{ ----- (2.6)}$$

## 2.2 Matrix Method of Contingency Analysis

This method makes use of bus impedance matrix associated with both base case system and the system modified by either line removals or additions. Z-matrix of a system can be obtained by inverting the bus admittance matrix or it can be constructed by using available algorithms. The fundamental approach to contingency analysis using z-matrix method is to inject a fictitious current in to one of the buses associated with the element to be removed, of such value that the current flow through the element equals the base case flow; all the other bus currents are set equal to zero. In effect, this procedure creates throughout the system a current flow pattern that will change in the same manner as the current flow pattern in the AC load flow solution when the element in question is removed.

This method is more accurate compared to DC load flow method and the results are comparable to those obtained using AC power flow.

## 2.3 Voltage Stability Index (L-Index) computation

This method is also based on load flow analysis and used for determining the voltage collapse proximity. The value of L-index ranges from 0 (no load condition) to 1 (Voltage collapse). The bus with highest L-index value will be the most vulnerable bus in the system. Calculation of L- index for a power system is briefly disused below.

Consider an N bus system with number of generators Ng. The voltages and currents are represented by equation

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \text{ -----(2.7)}$$

Where,

$I_G, I_L$  and  $V_G, V_L$  represent the complex current and voltage vectors at generator and load nodes.  
 $[Y_{GG}], [Y_{GL}], [Y_{LG}]$  and  $[Y_{LL}]$

They are corresponding partitioned portion of the network admittance matrix. Rearranging the equations,

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \text{ ----- (2.8)}$$

Where

$$F_{LG} = [Y_{LL}]^{-1} [Y_{LG}] \text{ -----}$$

(2.9) The L-index of the j th node is given by

The value of  $F_{ji}$  are obtained from matrix  $F_{LG}$  from equation (2.9)

$$L_j = \left| 1 - \sum_{i=1}^{N_g} F_{ji} \frac{V_i}{V_j} \angle (\theta_0 + \delta_i - \delta_j) \right| \text{ -----(2.10)}$$

## 2.4 Decoupled Load Flow

The Newton Raphson power flow equations in Jacobian form are represented by the following equation.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \quad \text{-----(2.11)}$$

$$\text{Where } H_{ij} = \frac{\partial P_i}{\partial \delta_j}; \quad N_{ij} = \frac{\partial P_i}{\partial |V_j|};$$

$$J_{ij} = \frac{\partial Q_i}{\partial \delta_j}; \quad L_{ij} = \frac{\partial Q_i}{\partial |V_j|}; \quad \text{-----(2.12)}$$

H, N, J and L are the sub-matrices of the Jacobian represented by Equation 2.12.

In a power system there is a strong dependence between injected real powers and bus voltage angles and between the injective reactive power and bus voltage magnitudes that is strong couplings between P and  $\delta$  variables and between Q and |V|. The coupling between Q and  $\delta$  and P and |V| is weak. Therefore the matrices N and J can be set to zero. Resulting linear equations reduce to

$$\Delta P_{(k-1)} = H_{(k-1)} \Delta \delta_k \quad \text{-----(2.13)}$$

$$\Delta Q_{(k-1)} = L_{(k-1)} \left[ \frac{\Delta |V|}{|V|} \right] \quad \text{-----(2.14)}$$

The elements of H and L are given in Equation 2.12.

## 2.5 Fast decoupled Load Flow

An important and useful property of power system is that the change in real power is primarily governed by the charges in the voltage angles, but not in voltage magnitudes. On the other hand, the charges in the reactive power are primarily influenced by the charges in voltage magnitudes, but not in the voltage angles. To see this, let us note the following facts:

1. Under normal steady state operation, the voltage magnitudes are all nearly equal to 1.0.
2. As the transmission lines are mostly reactive, the conductance's are quite small as compared to the susceptance,  
 $G_{ij} \ll B_{ij}$
3. Under normal steady state operation the angular differences among the bus voltages are quite small

$$(\theta_i - \theta_j \approx 0 \text{ (within } 5^\circ - 10^\circ))$$

4. The injected reactive power at any bus is always much less than the reactive power consumed by the elements connected to this bus when these elements are shorted to the ground

$$(Q_i \ll B_{ii} V_i^2)$$

With these facts at hand, let us re-visit the equations for Jacobian elements in Newton-Raphson (polar) method

$$P_i = \sum_{j=1}^n |V_{ij}| |V_i| |V_j| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = \sum_{j=1}^n |V_{ij}| |V_i| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(k)}}{\partial \delta_2^{(k)}} & \dots & \frac{\partial P_2^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial P_2^{(k)}}{\partial |V_2|} & \dots & \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial P_n^{(k)}}{\partial \delta_2^{(k)}} & \dots & \frac{\partial P_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial P_n^{(k)}}{\partial |V_2|} & \dots & \frac{\partial P_n^{(k)}}{\partial |V_n|} \\ \frac{\partial Q_2^{(k)}}{\partial \delta_2^{(k)}} & \dots & \frac{\partial Q_2^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} & \dots & \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_n^{(k)}}{\partial \delta_2^{(k)}} & \dots & \frac{\partial Q_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial Q_n^{(k)}}{\partial |V_2|} & \dots & \frac{\partial Q_n^{(k)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

The diagonal and off diagonal elements of J1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i}^n |Y_{ij}| |V_i| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial \delta_j} = - |Y_{ij}| |V_i| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

The diagonal and off diagonal elements of  $J_2$

$$\frac{\partial P_i^{(k)}}{\partial |V_i|} = 2|Y_{ii}| |V_i| \cos(\theta_{ii}) + \sum_{j \neq i}^n |Y_{ij}| |V_j| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i^{(k)}}{\partial |V_j|} = |Y_{ij}| |V_i| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

The diagonal and off diagonal elements of  $J_3$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i}^n |Y_{ij}| |V_i| |V_j| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial \delta_j} = - |Y_{ij}| |V_i| |V_j| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

The diagonal and off diagonal elements of  $J_4$

$$\frac{\partial Q_i^{(k)}}{\partial |V_i|} = -2|Y_{ii}| |V_i| \cos(\theta_{ii}) - \sum_{j \neq i}^n |Y_{ij}| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i^{(k)}}{\partial |V_j|} = -|Y_{ij}| |V_i| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

$$\frac{\partial P_i}{\partial V_i} \approx 0 \quad \text{and} \quad \frac{\partial P_i}{\partial V_j} \approx 0 \quad \implies J_2 \approx 0$$

$$\frac{\partial Q_i}{\partial \theta_i} \approx 0 \quad \text{and} \quad \frac{\partial Q_i}{\partial \theta_j} \approx 0 \quad \implies J_3 \approx 0$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\Delta P = J_1 \Delta \delta = \frac{\partial P_i}{\partial \delta_j} \Delta \delta$$

$$\Delta Q = J_4 \Delta |V| = \frac{\partial P_i}{\partial |V|} \Delta |V|$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i}^n |Y_{ij}| |V_i| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2 |Y_{ii}| \sin(\theta_{ii})$$

Replacing the first term of this above equation with  $-Q_i$

$$\begin{aligned} \frac{\partial P_i}{\partial \delta_i} &= -Q_i - |V_i|^2 |Y_{ii}| \sin(\theta_{ii}) \\ &= -Q_i - |V_i|^2 B_{ii} \end{aligned}$$

Where  $B_{ii} = |Y_{ii}| \sin \theta_{ii}$  is the imaginary part of the diagonal element of the admittance matrix.  $B_{ii}$  is the sum of the susceptance of all the elements incident to bus  $i$ . In a typical power system the self susceptance  $B_{ii} \gg Q_i$  and assuming  $|V_i|^2 \approx |V_i|$ , which yields

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii}$$

Under normal operation condition,  $\delta_i - \delta_j$  is quite small, thus assuming  $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ii}$  the off diagonal elements of  $J_1$  becomes,

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| |V_j| B_{ij}$$

Further simplification is obtained by assuming  $V_j \approx 1$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| B_{ij}$$

Similarly, the diagonal elements of  $J_4$  may be written as

$$\frac{\partial Q_i^{(k)}}{\partial |V_i|} = -|Y_{ii}| |V_i| \sin(\theta_{ii}) - \sum_{j=i}^n |Y_{ij}| |V_i| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Replacing the second term of this above equation with  $-Q_i$

$$\frac{\partial Q_i^{(k)}}{\partial |V_i|} = -|Y_{ii}| |V_i| \sin(\theta_{ii}) + Q_i$$

since  $B_{ii} = |Y_{ii}| \sin(\theta_{ii}) \gg Q_i$

$$\frac{\partial Q_i^{(k)}}{\partial |V_i|} = -|V_i| B_{ii}$$

assuming  $\theta_{ii} - \delta_i + \delta_j \approx \theta_{ij}$  results in,

$$\frac{\partial Q_i^{(k)}}{\partial |V_j|} = -|V_i| B_{ij}$$

Here,  $B'$  and  $B''$  are the imaginary part of bus admittance matrix  $Y_{Bus}$ , therefore in fast decoupled loadflow power algorithm, the successive voltage magnitude and phase angle changes are given by,

$$\Delta \delta = -[B']^{-1} \frac{\Delta P}{|V|}$$

$$\Delta |V| = -[B'']^{-1} \frac{\Delta Q}{|V|}$$

Thus it can be concluded that fast decoupled power flow solution requires the least time per iteration among all load flow techniques available, hence the power flow solution is obtained very rapidly, thus this technique is very useful in contingency analysis where numerous outage are to be simulated in a very rapid manner.

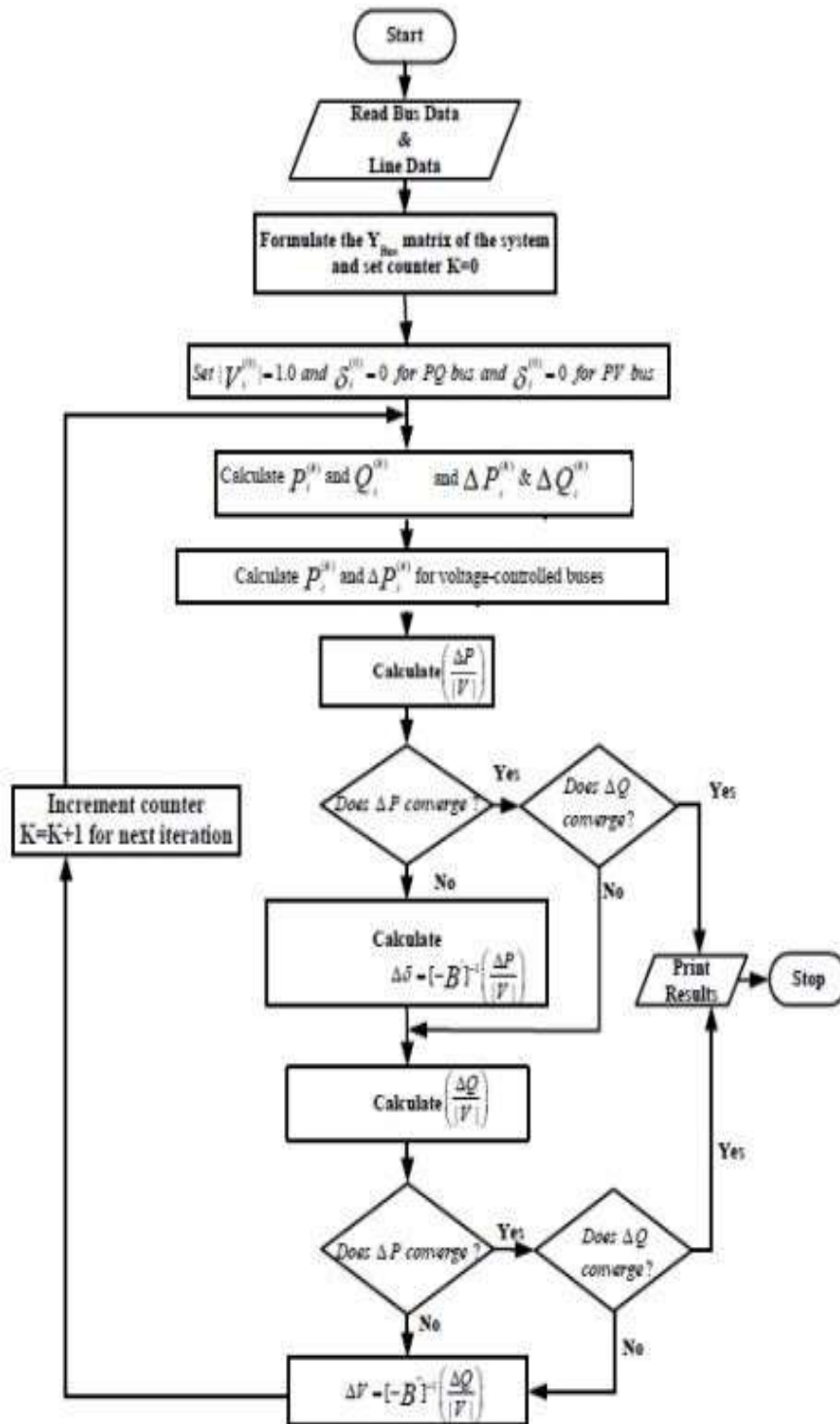


Fig 2.1 Flow chart for FDLF algorithm

### III. CONTINGENCY ANALYSIS TECHNIQUES

It is important to know which line or unit outage will render line flows or voltage to cross the limits. To find the effects of outages, contingency analysis techniques are employed. Contingency analysis models single failure events (i.e. one-line outage or one unit outages) or multiple equipment failure events (failure of multiple unit or lines or their combination) one after another unit all “credible outages” are considered. For each outage, all lines voltage in the network are checked against their respective limits. Figure 2.2 depicts a flow chart illustrating a simple method for carrying out a contingency analysis.

#### IV. CONTINGENCY RANKING

In descending order contingency ranking is obtained according to the value of a scalar index which is normally called as severity index or performance index (PI). The PI is a measurement of system-wide effect of a contingency event in the system. It is calculated for individual contingency in off line mode using the conventional load flow algorithm. Based on the obtained values contingencies are ranked in such a manner where highest value of PI is ranked first. Basically there are two types of performance indices that are of great use, i.e., active power index (PIP) and reactive power index (PIV). PIP reflects violation of line active power flow and can be calculated using the mathematical expression

$$PI_P = \sum_{i=1}^L \left( \frac{P_i}{P_{i\max}} \right)^2 \text{-----}(2.22)$$

Where, L = Total number of transmission lines that present in the system  $P_i$  = Active power flow in line i  $P_{i\max}$  = Maximum active power flow in line i

$$= \frac{|V_i||V_p|}{|Z|} - \frac{R|V_p|}{|Z|^2}$$

Z = Impedance of the line connecting buses i and p  
R = Resistance of the line connecting buses i and p

PIV reflects bus voltage magnitude violation and can be calculated by the mathematical expression

$$PI_V = \sum_{i=1}^{n-m-1} \left( \frac{2(V_i - V_{i\text{nom}})}{V_{i\max} - V_{i\min}} \right) \text{-----}(2.23)$$

Where, n = Total number of buses present in the system m = Total number of PV buses present in the system n-m-1 = Total number of load buses present in the system  $V_i$  = Voltage of bus i after load flow  $V_{i\text{nom}}$  = Nominal voltage at bus-i. Generally assumed as 1 pu.

$V_{i\min}$  = Minimum voltage limit. Generally assumed as 95% of  $V_{i\text{nom}}$ .  $V_{i\max}$  = Maximum voltage limit. Generally assumed as 105% of  $V_{i\text{nom}}$ .

#### V. CONTINGENCY SELECTION

The severe contingencies are then chosen from either of ranked list starting from the top and going down the list until predefined stopping criteria is reached. These are absolute fixed lists. The process of choosing a subset that containing severe contingencies is called contingency selection. This process consists of selecting the set of most probable contingencies, they need to be evaluated in terms of potential risk to the system.

#### VI. CONTINGENCY EVALUATION

At last in order of their severity the selected contingencies are ranked, till no violation of operating limits is observed.

#### VII. CONCLUSION

By calculating performance indices, i.e., active power performance index (PIP) and reactive power performance index (PIV) the contingency selection and contingency ranking are made in this paper. These two indices PIP and PIV were calculated for an IEEE 25 bus, 35 line test system. The contingency severity cases, i.e., single line outage and double line outage is accurately indicated by the numerical values of PIP and PIV respectively. In off line manner the indices are calculated for a single loading condition. From the obtained results it can be concluded that the calculation of performance indices gives a good measure about the severity of all the possible line contingencies occurring in the system.

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