

Concurrent Design of Input Shaping and Proportional plus Derivative Feedback Control

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Abstract: This study presented a system that exhibit flexible dynamics are widespread and present a very challenging control problem when their performance is pushed to the limit. If there is some knowledge of the flexible modes, then command signals can be generated to negate the detrimental dynamics. These vibration-reducing commands are dependent on the feedback controller gains because the gains influence the flexible modes. This paper presents a method for concurrently designing a PD feedback controller and a command generator so that performance is optimized. The design method takes into account limits on allowable overshoot, residual vibration, and actuator effort. Furthermore, the structure of the method allows a wide range of performance requirements, such as disturbance rejection, to be integrated into the design. Results demonstrate that a PD controller cannot achieve the same performance as a PD controller augmented with a command generator.

Key Word: Proportional plus Derivative (PD) Feedback Controller, Input Shaping Process, Command Enhanced Feedback (CEF) method, Transfer Function, Percent Overshoot, Actuator Effort, Residual Vibration, Settling time, Sensitivity.

I. INTRODUCTION

Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so, on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system. The vibration of flexible systems often limits speed and accuracy. If the system dynamics are known with some confidence, then there are several techniques for generating commands that will negate the system's flexible modes [1]. Input shaping has been implemented on a variety of systems. The performance of longreach manipulators [7-8] cranes [9] and coordinate measuring machines was improved with input shaping. In particular, input shaping has shown great promise when used in conjunction with feedback control on flexible robot arms. A combined input shaper and feedback controller was successfully implemented on a five-bar linkage manipulator in [10]. Magee and Book used input shaping in conjunction with feedback control to reduce the vibration of a small articulated robot mounted on the end of a long, slender beam [8]. Designing input shapers in the z-domain has proven to be easy and effective [11].

II. MATERIAL AND METHODS

The Modeling of this study start with the approach used to simultaneously determine the input shaper parameters and feedback gains is called the Command Enhanced Feedback (CEF) design method. The position control of a single mass is analyzed in order to demonstrate the CEF method. To summarize the Command Enhanced Feedback design process, equations are satisfied while minimizing the time of the final impulse in the input shaper. This modeling uses MATLAB Software. Simulation and Model-Based Design of controller are performed in Simulink part of MATLAB Software. Simulink provides a graphical editor, customizable block libraries, and solvers for modeling and simulating dynamic systems. It is integrated with MATLAB, enabling you to incorporate MATLAB algorithms into models and export simulation results to MATLAB for further analysis. The plot of kinematic analysis data and plot of Modeled equation is also compared in the MATLAB software. Furthermore, by concurrently designing the feedback gains and the input shaper, higher levels of performance can be achieved than by the traditional method of first designing the feedback gains and then designing the input shaper. Finally, a stability analysis and an investigation of the effect of disturbances have supported the value of the proposed method.

The modeling of equation for comparing the effectiveness of the CEF approach to PD control:

1. Overshoot
2. Settling Time
3. Effect of Disturbances on Actuator Effort
4. Sensitivity
5. Effect of Unmodeled Modes.

The Simulation analysis of the equation is performed in MATLAB software.

Comparison of Control Methods To evaluate the effectiveness of the CEF method, it is compared to standard PD control without input shaping. Each controller was designed for a system having a nominal mass of 1 kg, maximum actuator

effort of 200 N, and a maximum expected step size of 1 m. To aid in the controller evaluation, the following definition is used:

Where U_{tol} is the actuator limit, m is the nominal mass of the system, and L is the magnitude of the step change in position. Physically, α/L provides a quantitative measure of the ability of the system to respond to a step input. In general, a system with a large value of α/L will respond faster than a system with a small value of α/L . The parameters for the example system yield $\alpha/L = 200$. In order to make a more valid comparison of control systems, both control schemes are required to have a time constant of 0.116 s. The time constant of the system, defined as $(\zeta\omega_n)^{-1}$, gives an indication of the system's ability to reject disturbances. Note that requiring each system to have the same time constant is equivalent to requiring the two systems to have the same value of KD . However, the CEF method results in a larger effective spring () than PD control alone.

1. Overshoot- In order to evaluate the overshoot properties of the system, (3) is first solved for $Y(s)$. The resulting equation for $Y(s)$ is rewritten into the standard form of a damped harmonic oscillator subject to a step input of magnitude L . This yields the following expression:

$$Y(s) = G_{CL}(s)R(s) = \frac{L}{s} \frac{2\zeta\omega_n(s+z)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where

$$z = \frac{\omega_n}{2\zeta}$$

$$\omega_n = \sqrt{\frac{K_P}{m}}$$

$$\zeta = \frac{K_D}{2\sqrt{mK_P}}$$

An expression for the percent overshoot of the system as a function of the damping ratio, ζ can be obtain after (42) is converted to the time domain. This expression is given by:

$$M_p = 100e^{(-\zeta/\sqrt{1-\zeta^2})(\pi-\theta)}$$

Where

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{(1-2\zeta^2)/2\zeta}\right)$$

The percent overshoot for the system under PD control versus damping ratio is shown in figure 1. The graph shows that, even with a damping ratio of 1, the best achievable overshoot is 13.5 percent. The fact that a critically damped system exhibits overshoot may seem counterintuitive, but the zero dynamics in (42) affects the response of the system due to its close proximity to the location of the complex poles for high values of ζ .

Recall that the overshoot can be arbitrarily specified with the CEF method. For the purposes of this paper, a peak overshoot of 5 percent was chosen for the CEF controller. A larger overshoot would result in a faster rise time, but accurate positioning and a fast settling time were assumed to be very important.

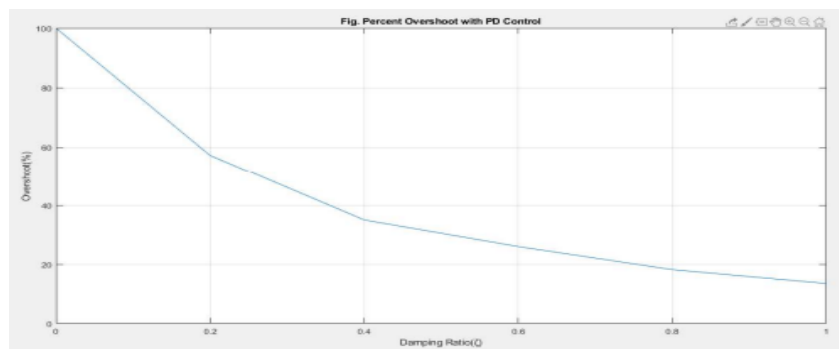


Figure 1: Percentage Overshoot with PD Control

2. Settling Time - Settling time is a primary concern for most industrial applications, as a faster settling time will result in increased accuracy and higher throughput. For this work, the system is defined to be settled when the residual vibration falls below settling time that can be attained with only PD control is plotted as a function of damping ratio in Fig. 2 for α/L value of 200. The following equation was used to generate the curve of PD settling time in Fig. 2

$$t_s = \frac{\ln(0.05\sqrt{1-\zeta^2})}{-\zeta\omega_n}$$

Referring to Fig. 2, the fastest settling time that can be realized with PD control alone is 0.395 s. This settling time requires a damping ratio of 0.75, which will yield an overshoot of 19.4 percent. As a side note, the settling time defined by reduce disturbance-induced vibration to a 5 percent level.

The settling time is equal to the time of the final input. The final impulse defines the settling time because the residual vibration to below a given value (5 percent in this case) at the time of Figure 2 shows the best achievable settling time with CEF control as a function of damping ratio for $\alpha/L = 200$. The minimum settling time with CEF control is 0.153 s, about 2.6 times faster than with PD control alone! Although Fig. 2 same basic result occurs for other values α/L .

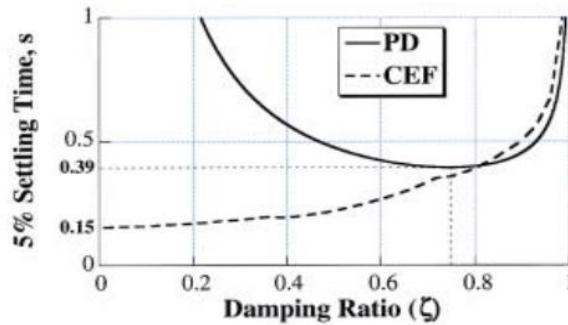


Figure 2: Effect of damping on settling Time

3. Effect of Disturbances on Actuator Effort - External disturbances introduced into the system will result in some amount of actuator effort. Disturbances that directly affect the plant input are shown as $D(s)$ in Fig.2. Measurement noise, or other similar error sources, could also be introduced after the plant, and they are represented by $W(s)$ in Fig. 2. The effect of both types of disturbances must be considered to ensure that they do not result in actuator saturation when added to the actuator effort resulting from the reference command. The effect of $D(s)$ on input magnitude can be analyzed with the following transfer function, assuming $W(s)$ is zero and $Y_d(s)$ is not changing:

$$\frac{U(s)}{D(s)} = \frac{1}{1 + G_P(s)G_{PD}(s)} = \frac{ms^2}{ms^2 + K_D s + K_P}$$

Figure 3 shows the magnitude of (49) for frequencies ranging from 0 rad/s to 1000 rad/s. The corner frequency of the system under PD control is 11.5 rad/s, and the corner frequency of the system under CEF control is 17.9 rad/s. Figure 5 represents the amplification between $D(s)$ disturbances and actuator effort. To find the magnitude of actuator effort resulting from a type $D(s)$ disturbance, the magnitude of the disturbance is added to the magnitude from Fig.3 corresponding to the frequency of the disturbance. Figure3 shows that, at low frequencies, the amplification of type $D(s)$ disturbances is almost 10 dB less with CEF control than with PD control. An expression similar to was obtained for disturbances of type $W(s)$ and is given by:

$$\frac{U(s)}{W(s)} = \frac{G_{PD}(s)}{1 + G_{PD}(s)G_P(s)} = \frac{ms^2(K_D s + K_P)}{ms^2 + K_D s + K_P}$$

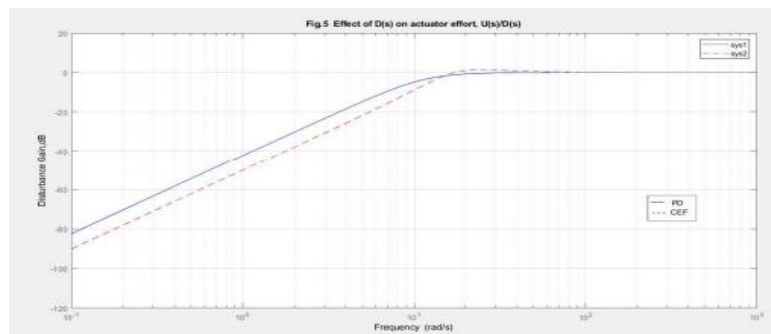


Figure 3: Effect of $D(s)$ on Actuator Effort $U(s) / D(s)$

The magnitude of is given in Fig. 3 and indicates that $W(s)$ disturbances are amplified by about the same amount with either control method. Once the magnitude of actuator effort resulting from each type of disturbance is calculated, U_{tol} can be adjusted to ensure that the disturbances will not cause actuator saturation.

Notice that in Fig. 3 and 4, the CEF control system results in slightly larger disturbance amplification than the PD control system near the corner frequency. However, the corner frequency corresponds to the natural frequency of the closed-loop system. If it is known that disturbances will be introduced near the corner frequency, then the gains would be changed to move the system poles away from this region.

4. Sensitivity - The sensitivity of a system to disturbances of types $D(s)$ and $W(s)$ is a good indicator of system performance. Low sensitivity values are desired, as this indicates that disturbances will have little effect on both the output of the system and the magnitude of the error signal. The sensitivity of the response to disturbances of type $D(s)$ is given by:

$$\frac{Y(s)}{D(s)} = \frac{G_P}{1 + G_{PD}G_P} = \frac{1}{ms^2 + K_D s + K_P}$$

This transfer function for both control methods is shown in Fig. 5. The CEF control method is less sensitive to low frequency disturbances and exhibits approximately the same sensitivity to high frequencies.

The sensitivity transfer function relating disturbances of type $W(s)$ to system response is equivalent to the closed-loop transfer function defined by (3). This closed-loop transfer function is shown in Fig. 6. Both control methods have similar performance at high frequencies, which are usually of greatest concern with disturbances of type $W(s)$. At low frequencies, the CEF controller produces a curve with a slightly larger bandwidth. If the increase in bandwidth has a significant effect on the system response due to errors such as low frequency measurement noise, adjustments can be easily made in the CEF design algorithm to reduce the bandwidth.

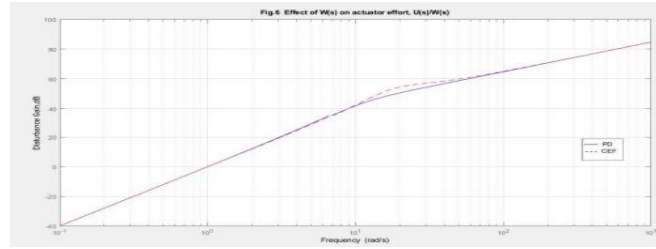


Figure 4: Effect of $W(s)$ on Actuator Effort $U(s)/W(s)$

Error Sensitivity – The final measure of system sensitivity relates the magnitude of the error signal, $E(s)$, to the reference command, $R(s)$. This sensitivity should be very low, as a small error signal typically allows for larger gains and improved performance. Furthermore, the error sensitivity is an indicator of the general robustness of the closed loop system. The error sensitivity transfer function is given by:

$$\frac{E(s)}{R(s)} = \frac{1}{1+G_P D G_P} = \frac{ms^2}{ms^2 + K_D s + K_P}$$

Figure 7 indicates that the CEF control system will result in error signals of smaller magnitude than the PD controller for low frequency inputs, while at high frequencies the two controllers produce error signals of similar magnitude.

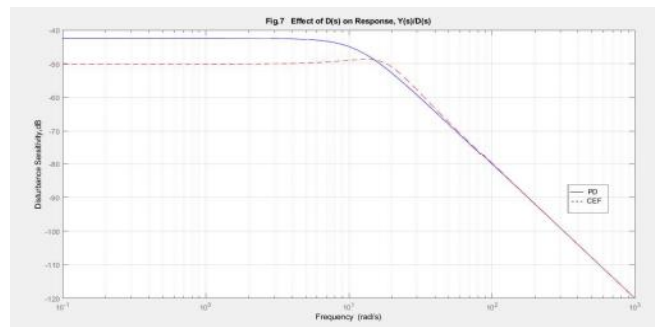


Figure 5: Effect of $D(s)$ on Response $Y(s) / D(s)$

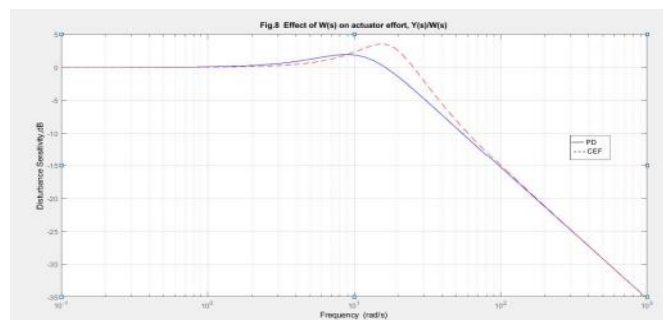


Figure 6: Effect of $W(s)$ on Response $Y(s) / W(s)$

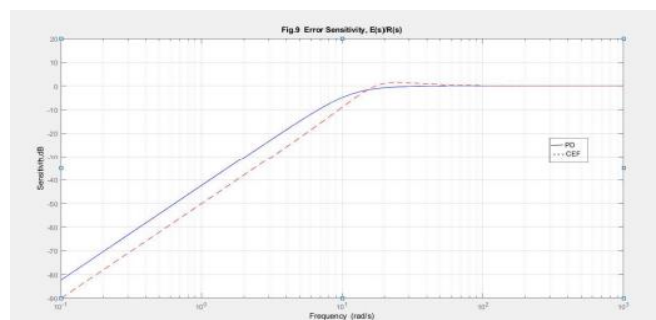


Figure 7: Error Sensitivity $E(s) / R(s)$

5. Effect of Unmodeled High Modes - To analyze the effect of unmodeled high modes on the system performance with each type of control, an undamped flexible mode is attached to the baseline mass. By unmodeled mode, we mean a high

mode that is not taken into account when designing the PD and CEF control systems. Modeling the additional mode as a spring-mass system characterized by k_2 and m_2 , the transfer function of the physical plant becomes:

$$G_p(s) = \frac{m_2 s^2 + k_2}{s^2 [m m_2 s^2 + (m + m_2) k_2]}$$

The overall closed-loop transfer function including the unmodeled mode then becomes:

$$G_{CL}(s) = \frac{m_2 K_D s^3 + m_2 K_P s^2 + K_2 K_D s + K_2 K_P}{m m_2 s^4 + m_2 K_D s^3 + [(m + m_2) k_2 + m_2 K_P] s^2 + k_2 K_D s + k_2 K_P}$$

The effect of the unmodeled mode on the stability of the system is a primary concern. The system should be stable over a wide range of reasonable parameters of both the baseline mass and the unmodeled high mode. Using the Routh stability criteria, the first column of the Routh array has values of:

$$\begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} m m_2 \\ m_2 K_D \\ m_2 (k_2 + K_P) \\ \frac{k_2^2 K_D}{k_2 + K_P} \\ k_2 K_P \end{bmatrix}$$

This section has compared CEF control to optimized PD control without input shaping. Hopefully, the reader can see that it is an unfair comparison—the CEF control is much better in all respects, except for phase margin. Another comparison can be made between CEF control and the optimized PD control with input shaping added after the PD gains have been chosen. PD control with input shaping achieves performance levels closer to the CEF higher proportional gain of the CEF control allows it to move the system faster. This is another key result concurrently designing the feedback gains and input shaper, (CEF algorithm), provides better performance than the traditional and then designing the input shaper based on the closed loop performance.

III.RESULT

The response of the mass to a 1m step input under both types of control is shown in Fig. 1, along with the reference commands. The benefits of using the CEF design method, in terms of overshoot and settling time are very apparent. Figure 1 shows a potential advantage of the CEF method when the maximum overshoot value is set equal to the residual vibration limit. Because the maximum overshoot for the CEF method is set to 5 percent, the settling time of the system under CEF control is equal to the rise time, approximately 0.153 s. This is not the case when the peak overshoot is allowed to be above the value used to determine the settling time and it is consequently not stated as a general result. However, for many control system designs this type of response will be observed.

In addition to the response of the closed-loop system to reference commands, the disturbance rejection properties of each control scheme also provide a good basis for comparison. The effect of an impulse disturbance of type D(s) introduced at 1s is shown in Fig. 2. Because both controllers provide the same time constant, they are equally capable of rejecting type D(s) disturbances. However, the higher proportional gain of the CEF control allows it to start suppressing the disturbance sooner than the PD control. Therefore, although settling time from a disturbance is the same, the CEF control results in a smaller excursion when a disturbance occurs.

Type W(s) disturbances will affect the response of the mass in the manner shown by Fig. 3. The W(s) disturbance used to generate Fig. 3 was a sinusoidal input of amplitude 0.05 and frequency 115 rad/s. As predicted by Fig. 3, the steady-state responses of the two closed loop systems to the W(s) disturbance are identical at a frequency of 115 rad/s.

Notice from Table 4.1 that the optimization routine actually solves for the proportional and derivative gains divided by the nominal system mass. This implies that, if the nominal value of mass varies from the expected value, new values of K_P and K_D can be calculated. Also, if the nominal mass value varies, then the maximum step size should be changed accordingly to achieve the same value of a/L used in the design. If these two steps are taken, then the same control scheme can be used under a wide range of different operating conditions. However, if the nominal mass value changes permanently from the value specified during the design process, then a new set of control parameters should be calculated to ensure that optimal performance is achieved.

Table 4.1 Control method parameters

	PD only	CEF
K_P/m	132.2	321.5
K_D/m	17.24	17.24
ζ	0.75	0.48
τ , second	0.116	0.116
Overshoot, %	19.4	5
t_s , second	0.395	0.153

The results shown in this section and in Section 4 clearly demonstrate that the CEF control out performs PD control

for the problem considered here. However, there is still the question of how CEF control compares to the traditional input shaping process where the feedback gains are chosen first and then the input shaper is designed using the resulting dynamics. The relative improvement with this new approach cannot be stated definitively because the answer depends on how the PD gains are chosen in the traditional input shaping approach. If, by some chance, PD gains were chosen to be the exact values determined through the CEF approach, then the traditional method could yield equivalent performance (provided that the optimal shaper was then designed in the subsequent step). However, the traditional approach does not have any mechanism for finding the optimal gain values, other than through a trial and error process that includes designing the complimentary input shaper and evaluating the resulting performance.

Simulink Model for PD and CEF Control action -

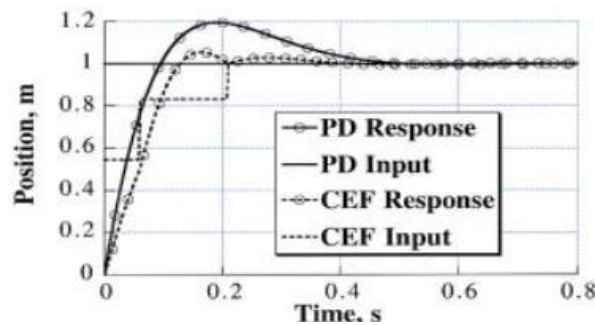
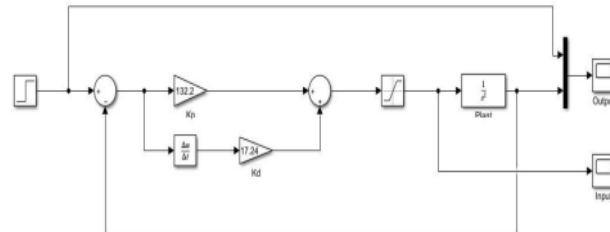


Figure 1: Response to PD and CEF Control Action

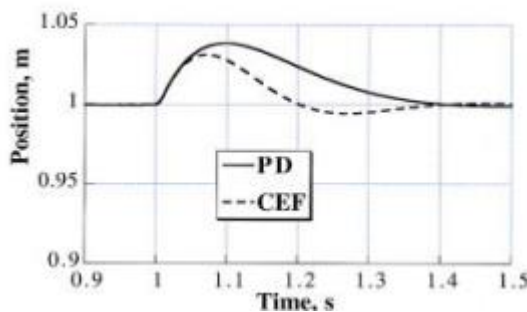


Figure 2: Response to $D(s)$ Disturbance

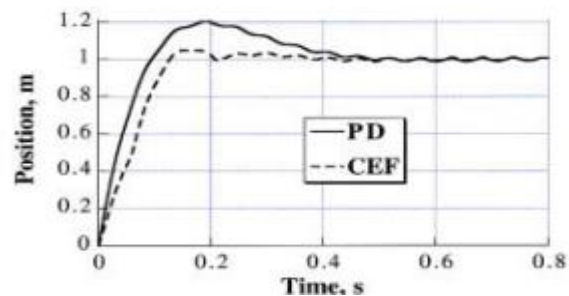


Figure 3: Response to $W(s)$ Disturbance

IV.DISCUSSION

The traditional approach would usually choose the gains so that the unshaped response would be fairly well damped. In this case, the proportional gain would be smaller or the derivative gain would be larger than in the CEF case, and the CEF controller would produce a faster response. Therefore, the traditional sequential input shaping design approach would, at best, equal the CEF control, but for practical reasons it would not perform as well.

The cost of using the CEF approach, as compared to the traditional approach, is the possibility of increased computational complexity. On the other hand, the traditional approach might require a large number of trial-and-error attempts to obtain near-optimal performance. The end result being that the CEF approach could save the designer a significant amount of time.

V.CONCLUSION

A method has been presented for concurrently designing a PD feedback controller and an input shaper so that performance is optimized. The design method takes into account limits on allowable overshoot, residual vibration, and actuator effort. Furthermore, the structure of the method allows a wide range of performance requirements, such as disturbance rejection, to be integrated into the design. The results clearly indicate that PD feedback control enhanced with input shaping provides better performance than PD control alone.

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